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Solutions of Separation, Simple Union and Simple Intersection Equations

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ABSTRACT

Separation, simple union and simple intersection equations of sets are particular set equations with some special properties. The formal definition of separation, simple union and simple intersection equations of sets is given in introduction section. In this research, we study properties of solutions of separation, simple union and simple intersection equations of sets. We use some set operations and De Morgan's law to investigate some facts of separation, simple union and simple intersection equations of sets. Further, we study the relation between separated sets and separation equation of sets. In addition, we investigate necessary and sufficient condition for solutions of separation equation.

Keywords: Disjoint, Operation, Separated Sets, Sigma algebra, Solution, Topology

I. INTRODUCTION

The equation $(A \cup B)^c \cup (A \cup C)^c = (A \cup C)^c \cup C$ is called separation equation of a set C with respect to A and B. The equations $A \cup Z = B$ and $A \cap Y = B$ are called simple union and simple intersection equations, respectively. Solving for unknown set from set equations is not easy in general. We are familiar with sets and set operations. However, solution method for set equations is not known except for particular cases. For instance, given two sets A and B, solving for Z from the set equation $A \cup Z = B$ is not trivial. One may find one particular solution by chance, but what is the general solution? This just to show that solving for unknown set from a set equation is not obvious in general case. In literature, basic set theory concepts are available but solution methods for set equations are not available in advance. For this reason, we are interested to study some simple set equations. In this research, we study some special class of set equations, namely, separation equations, simple union and simple intersection equations. Further, we study solution sets of these special set equations. Moreover, we construct necessary and sufficient condition for a set to be a solution of separation equation. Finally, we construct the so called separation topology and separation sigma algebra from solution set of separation equations. Basic set operations and De Morgan's law are explained in detail see [1]. However, solution of set equations is not introduced in [1]. For this reason, we are interested to introduce solution of some special set equations, namely, separation, simple union and simple intersection equations. This indicates that this research work is an original work on solution of separation, simple union and simple intersection equations.

II. PRELIMINARIES

Definition 1 [1] Set is a collection(family) of well defined objects. An object in a set is called an element of a set. A set E is called subset of X if all elements of E belong to X. For a set X, its power set, denoted by P(X) or 2^X, is the collection of all subsets of X .

Remark 2 [1] If $E \subset X$ and the set X is a universal set, we denote the complement of E in X by $E^c = X \setminus E$. Let I be an index set and $\mathcal{C} = \{E_\alpha \subset X: \alpha \in I\}$. Then

1. $\mathcal{C}$ is disjoint if $E_\alpha \cap E_\beta = \emptyset$ for all $\alpha, \beta \in I$ and $\alpha \neq \beta$.
2. De Morgan's laws state that $(\bigcap_{\alpha\in I} E_\alpha)^c = \bigcup_{\alpha\in I} E_\alpha^c$ and $(\bigcup_{\alpha\in I} E_\alpha)^c = \bigcap_{\alpha\in I} E_\alpha^c$.

Definition 3 [1] A σ-algebra on a set X is a collection τ of subsets of X such that:

1. $\emptyset, X \in \tau$,
2. if $A \in \tau$, then $A^c \in \tau$,
3. if $A_i \in \tau$ for $i \in \mathbb{N}$, then $\bigcup_{i=1}^{\infty} A_i \in \tau$.

Remark 4 [2] Let us consider the following basic properties of set operations.

1. For all subsets A , B of X , $A \cap B = B \cap A$ and $A \cup B = B \cup A$ [commutativity of the intersection and union operators].
2. For all subsets A, B and C of X,
$(A \cap B) \cap C = A \cap (B \cap C)$ and
$(A \cup B) \cup C = A \cup (B \cup C)$ [Associativity of the intersection and union operators].
3. For all subsets A, B and C of X,
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ [Distributivity of union over intersection] and
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ [Distributivity of intersection over union].

Definition 5 [3, 4, 5, 6] A topology on a set X is a collection τ of subsets of X, called the open sets, satisfying:

1. Any union of elements of τ belongs to τ,
2. Any finite intersection of subsets of τ belongs to τ,
3. $\emptyset$ and X belong to τ.

A pair (X, τ) is called a topological space. Note that every member of a topology on X is an open set of X. A subset of X is called a closed set of X if its complement is an open set of X. Closure of a subset A of X is the intersection of all closed sets of X containing A. Closure of A is denoted by $\overline{A}$.

Definition 6 Two sets A and B are said to be separated sets or set A is separated from set B if

$$A \cap \overline{B} = \emptyset \text{ and } B \cap \overline{A} = \emptyset.$$

Definition 7 Two sets A and B are said to be disjoint sets if

$$A \cap B = \emptyset.$$

Definition 8 A set C is called separated set from sets A and B if C is separated set from A and C is separated set from B.

Definition 9 The equation $(A \cup B)^c \cup (A \cup D)^c = (A \cup D)^c \cup D$ is called separation equation of a set C with respect to A and B if $C = D$. We denote separation equation of a set C with respect to A and B by $Sep(C = A \wedge B)$.

Definition 10 A set equation is an equation which contains one or more sets and set operations.

Example 11 Let $X = \{1,2,3,4\}$ be a universal set. Let $A = \{2,4\}$ and $B = \{1,2,3,4\}$. Then solve for Z and Y from the following set equations.

1. $A \cup Z = B$ 2. $A \cap Y = B$

This example shows that Z and Y need not be unique sets in general case. What are solutions of $A \cup Z = B$ and $A \cap Y = B$ whenever A and B are given sets? The equations $A \cup Z = B$ and $A \cap Y = B$ are called simple union and simple intersection equations, respectively.

III.RESEARCH QUESTIONS

What is C such that $Sep(C = A \wedge B)$?

What are necessary and sufficient conditions for C to be a solution of $Sep(C = A \wedge B)$?

What are sets Z and Y such that $A \cup Z = B$ and $A \cap Y = B$, respectively?

Some trivial facts for separation equations are given below.

Example 12 $Sep(\emptyset = A \wedge B)$ and

$Sep((A \cup B)^c = A \wedge B)$.

If $X$is universal set, then $Sep(X = A \wedge B)$ if and only if both A and B are empty sets.

Example 13 Let $X = \{a, b, c\}$, $A = \{a, b\}$ and $B = \{b\}$. Then find solutions of $Sep(D = A \wedge B)$. The answer is $D = \emptyset$ or $D = \{c\}$.

IV. OBJECTIVES

The general objective of this study is to solve separation equation, simple union equation and simple intersection equation of sets.

Specific objectives of this study are

1. To study the relation between separation equation of sets and separated sets.
2. To derive necessary and sufficient conditions for a set to be solution of separation equation of sets.
3. To construct separation topology and separation sigma algebra from solution set of separation equation.

V. METHODOLOGY

In this research, we use basic set operations and De Morgan's laws to solve separation equation, simple union equation and simple intersection equation of sets. Further, we use topology and sigma algebra axioms to construct separation topology and separation sigma algebra. Necessary and sufficient condition for a set to be solution of separation equation is obtained from the associated separation equation.

VI. RESULTS

We constructed solution for separation equation, simple union equation and simple intersection equation of sets.

Let us consider the simple union equation of sets. For two given sets A and B, simple union equation of A and B is given by $A \cup Z = B$, where Z is unknown subset of a universal set X to be solved.

Theorem 14 $Z = (B \cap A^c) \cup (A \cap H)$ is solution of $A \cup Z = B$ whenever $A \subset B$, where H any subset of a universal set X.

From this theorem observe that the solution need not be unique. For instance, $Z = B$ and $Z = B \cap A^c$ are solutions of $A \cup Z = B$.

For two given sets A and B, simple intersection equation of A and B is given by $A \cap Z = B$, where Z is unknown subset of a universal set X to be solved.

Theorem 15 $Z = (B \cup A^c) \cap (A \cup H)$ is solution of $A \cap Z = B$ whenever $B \subset A$, where H any subset of a universal set X.

From this theorem observe that the solution need not be unique. For instance, $Z = B$ and $Z = B \cup A^c$ are solutions of $A \cap Z = B$.

Theorem 16 Suppose that X is a universal set. For any subsets A, B and C of X,

$(A \cup B)^c \cup (A \cup C)^c = (A \cup C)^c \cup C$ if and only if $(A \cup B) \cap C = \emptyset$.

From this theorem observe that $Sep(C = A \wedge B)$ if and only if $C \subset (A \cup B)^c$ if and only if $C \cap A = \emptyset$ and $C \cap B = \emptyset$.

Therefore, solution set of $Sep(C = A \wedge B)$ is given by $S.S_{Sep(C=A\wedge B)} = \{C: C \subset (A \cup B)^c\}$. $S.S_{Sep(C=A\wedge B)}$ is the set of all solutions of $Sep(C = A \wedge B)$.

From the following figure, note that C is solution of the separation equation of A and B.

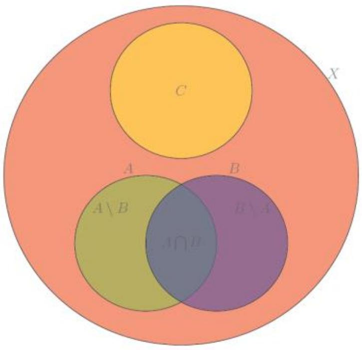

Figure 1: Solution of separation equation of A and B

Theorem 17 $S.S_{Sep(C=A\wedge B)}$ is topology on $(A \cup B)^c$. This topology is called separation topology.

Theorem 18 $S.S_{Sep(C=A\wedge B)}$ is sigma algebra on $(A \cup B)^c$ if and only if for all $D \in S.S_{Sep(C=A\wedge B)}$, $D^c \in S.S_{Sep(C=A\wedge B)}$. This sigma algebra is called separation sigma algebra.

VII. DISCUSSION

Necessary and sufficient conditions for a set C to be solution of separation equation of A and B are $A \cap C = \emptyset$ and $B \cap C = \emptyset$. Another interesting property is that any set which is separated from both A and B is solution of separation equation of A and B. This shows that solution set of separation equation of A and B contains all sets which are separated from both A and B. Intersection and union of solutions of separation equation of A and B are also solution of separation equation of A and B. Empty set and the complement of the union of A and B are solutions of separation equation of A and B. This shows that the solution set of separation equation of A and B is topology on the complement of the union of A and B. This topology is called separation topology.

The solution set of separation equation A and B is sigma algebra on the complement of the union of A and B if and only if the complement of every element of the solution set of separation equation A and B belongs to the solution set of separation equation A and B. We constructed solution method for simple union and simple intersection equations of A and B.

$Z = (B \cap A^c) \cup (A \cap H)$ is solution of simple union equation $A \cup Z = B$ whenever $A \subset B$, where H any subset of a universal set X.

$Z = (B \cup A^c) \cap (A \cup H)$ is solution of simple intersection equation $A \cap Z = B$ whenever $B \subset A$, where H any subset of a universal set X.

VIII. CONCLUSION

In this research, we have formed three special set equations, namely, separation, simple union and simple intersection equations of two sets. We obtained some solutions of separation, simple union and simple intersection equations of two sets. It is shown that solution set of a separation equation forms separation topology. Separation sigma algebra is constructed from solution set of separation equations. Necessary and sufficient condition for a set to be solution of separation equation is obtained. Moreover, we have shown all separated sets from A and B are solutions of separation equation of A and B. Since literature on set equations is not available in advance, further study on set equations is recommended for researchers. System of set equations is more difficult to solve in general. Study on system set equations must be carried out by researchers.

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