Gyanshauryam, International Scientific Refereed Research Journal
© 2020 GISRRJ | Volume 3 | Issue 5 | ISSN : 2582-0095

OPERATIONS ON THE FAMILY OF FUZZY SETS OF A SET AND ITS PROPERTIES THROUGH TOPOLOGY

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Article Info
Volume 3 Issue 5
Page Number: 93-99
Publication Issue :
September-October-2020
Article History
Accepted : 15 Oct 2020
Published : 26 Oct 2020

ABSTRACT

Generalized fuzzy topological space was introduced THROUG Mathematics Subject Classification: 54A40. The present paper is aimed to describe operations on the family of fuzzy sets of a set and discuss its properties.

Keywords : Generalized Fuzzy Topology, Γ–Fuzzy Open Set, Γ–Fuzzy Interior, Γ–Fuzzy Closure

1. Introduction and Preliminaries- Let X be a nonempty set and $F = \{\lambda \mid \lambda : X \to [0, 1]\}$ be the family of all fuzzy sets defined on X. Let $\gamma : F \to F$ be a function such that $\lambda \leq \mu$ implies that $\gamma(\lambda) \leq \gamma(\mu)$ for every $\lambda, \mu \in F$. That is, each γ is a monotonic function defined on F. We will denote the collection of all monotonic functions defined on F by $\Gamma(F)$ or simply Γ. Let $\gamma \in \Gamma$. A fuzzy set $\lambda \in F$ is said to be *$\gamma-$fuzzy open* [3] if $\lambda \leq \gamma(\lambda)$. Clearly, $\bar{0}$, the null fuzzy set is $\gamma-$fuzzy open. In [3], it is established that the arbitrary union of $\gamma-$fuzzy open sets is again a $\gamma-$fuzzy open set. A subfamily G of F is called a *generalized fuzzy topology* (GFT) [3] if $\bar{0} \in G$ and $\bigvee\{\lambda\alpha \mid \alpha \in \Delta\} \in G$ whenever $\lambda\alpha \in G$ for every $\alpha \in \Delta$. If $\gamma \in \Gamma$, it follows that A, the family of all $\gamma-$fuzzy open sets is a generalized fuzzy topology. For $\lambda \in F$, the *$\gamma-$interior* of λ, denoted by $i\gamma(\lambda)$, is given by $i\gamma(\lambda) = \bigvee\{v \in A \mid v \leq \lambda\}$. Moreover, in [1], it is established that for all $\lambda \in F$, $i\gamma(\lambda) \leq \lambda$, $i\gamma i\gamma(\lambda) = i\gamma(\lambda)$ and $\lambda \in A$ if and only if $\lambda = i\gamma(\lambda)$. A fuzzy set $\lambda \in F$ is said to be a *$\gamma-$fuzzy closed set* if $\bar{1} - \lambda$ is a $\gamma-$fuzzy open set. The intersection of all $\gamma-$fuzzy closed sets containing $\lambda \in F$ is called the *$\gamma-$closure* of λ. It is denoted by $c\gamma(\lambda)$ and is given by $c\gamma(\lambda) = \bigwedge\{\mu \mid \bar{1} - \mu \in A, \lambda \leq \mu\}$. In [1], it is established that $c\gamma(\lambda) = \bar{1} - i\gamma(\bar{1} - \lambda)$ for all $\lambda \in F$. A *fuzzy point* [4] $x\alpha$, with support $x \in X$ and value $0 < \alpha \leq 1$ is defined by $x\alpha(y) = \alpha$, if $y = x$ and $x\alpha(y) = 0$, if $y \leq = x$. Again, for $\lambda \in F$, we say that $x\alpha \in \lambda$ if $\alpha \leq \lambda(x)$. Two fuzzy sets λ and β are said to be *quasi-coincident* [4], denoted by

$\lambda q\beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ [4]. Two fuzzy sets λ and β are not quasi-coincident denoted by $\lambda\tilde{}q\beta$, if $\lambda(x)+\beta(x) \leq 1$ for all $x \in X$. Clearly, λ is a $\gamma-$fuzzy open set containing a point $x\alpha$ if and only if $x\alpha q\lambda$, and $\lambda \leq \beta$ if and only if $\lambda\tilde{}q(1 - \beta)$. For definitions not given here, refer [2].

2. Enlarging and quasi-Enlarging operations

Let X be a nonempty set and $\gamma \in \Gamma$. Let us agree in calling *operation*, any element of Γ. An operation $\gamma \in \Gamma$ is said to be *enlarging* if $\lambda \leq \gamma(\lambda)$ for every $\lambda \in F$. If $B \subset F$, then $\gamma \in \Gamma$ is said to be $B-$*enlarging* if $\lambda \leq \gamma(\lambda)$ for every $\lambda \in B$. We will denote the family of all enlarging operations by Γe and the family of all $B-$enlarging operations by ΓB. The easy proof of the following Theorem 2.1 is omitted.

Theorem 2.1.

Let X be a nonempty set and F be the family of all fuzzy sets defined on X. If $C \subset B \subset F$, then $\Gamma B \subset \Gamma C$. $\Gamma e = \Gamma B$, if $B = F$. An operation $\gamma \in \Gamma$, is said to be *quasi-enlarging* (QE) if $\gamma(\lambda) \leq \gamma(\lambda \wedge \gamma(\lambda))$ for every $\lambda \in F$. An operation $\gamma \in \Gamma$, is said to be *weakly quasienlarging*(WQE) if $\lambda \wedge \gamma(\lambda) \leq \gamma(\lambda \wedge \gamma(\lambda))$ for every $\lambda \in F$.If $\gamma \in \Gamma e$, then $\lambda \wedge \gamma(\lambda) = \lambda$ for every $\lambda \in F$ and so γ is quasi-enlarging. If γ is defined by $\gamma(\lambda) = \beta$ for every $\lambda \in F$, then also γ is quasi-enlarging. If $\gamma \in \Gamma$ is quasienlarging, then it is weakly quasi-enlarging, since $\lambda\wedge\gamma(\lambda) \leq \gamma(\lambda) \leq \gamma(\lambda\wedge\gamma(\lambda))$. The following Example 2.2 shows that a weakly quasi-enlarging operation need not be a quasi-enlarging operation.

Example 2.2.

Let $X = \{x, y, z\}$. Define $\gamma : F \rightarrow F$, by $\gamma(\lambda) = \bar{0}$, if $\lambda = \bar{0}$; $\gamma(\lambda) = \chi\{y\}$, if $\lambda \leq \chi\{x\}$; $\gamma(\lambda) = \chi\{z\}$, if $\lambda \leq \chi\{z\}$ and $\gamma(\lambda) = \bar{1}$ if otherwise. Then, $\lambda \wedge \gamma(\lambda) = \bar{0}$, if $\lambda = \bar{0}$; $\lambda \wedge \gamma(\lambda) = \bar{0}$, if $\lambda \leq \chi\{x\}$; $\lambda \wedge \gamma(\lambda) \leq \chi\{z\}$, if $\lambda \leq \chi\{z\}$ and $\lambda \wedge \gamma(\lambda) = \lambda$ if otherwise. Therefore, $\gamma(\lambda \wedge \gamma(\lambda)) = \bar{0}$, if $\lambda = \bar{0}$; $\gamma(\lambda \wedge \gamma(\lambda)) = \bar{0}$, if $\lambda \leq \chi\{x\}$; $\gamma(\lambda \wedge \gamma(\lambda)) = \chi\{z\}$, if $\lambda \leq \chi\{z\}$ and $\lambda \wedge \gamma(\lambda) = \bar{1}$, if otherwise and so it follows that γ is a weakly quasi-enlarging operator. If $\lambda = \chi\{x\}$, then $\gamma(\lambda) = \chi\{y\}$ but $\gamma(\lambda \wedge \gamma(\lambda)) = \gamma(\bar{0}) = \bar{0}$and so γ is not a quasi-enlarging operator. If $\gamma 1$, $\gamma 2 \in \Gamma$, then the composition $\gamma 1 \circ \gamma 2$ of the two operations $\gamma 1$ and $\gamma 2$ is again an operation and we write $\gamma 1\gamma 2$ instead of $\gamma 1 \circ \gamma 2$. The following Theorem 2.3 shows that the composition of enlarging operators is again an enlarging operator and Theorem 2.5 below gives a property of quasi-enlarging operators.

Theorem 2.3.

Let X be a nonempty set and F be the family of all fuzzy sets defined on X. If $B \subset F$, and $\gamma 1$ and $\gamma 2$ are $B-$enlarging, then $\gamma 1\gamma 2$ is also $B-$enlarging.

Proof.

Suppose $\lambda \in B$. Then $\lambda \leq \gamma 1(\lambda)$ and $\lambda \leq \gamma 2(\lambda)$. Now, $\lambda \leq \gamma 1(\gamma 2(\lambda))$, since $\gamma 1 \in \Gamma$. Therefore, $\gamma 1\gamma 2$ is $B-$enlarging.

Corollary 2.4.

If $\gamma 1, \gamma 2 \in \Gamma e$, then $\gamma 1\gamma 2 \in \Gamma e$.

Theorem 2.5.

Let X be a nonempty set, F be the family of all fuzzy sets defined on X and $B \subset F$. If $\gamma \in \Gamma$ is quasi-enlarging, $\{\gamma(\lambda) \mid \lambda \in F\} \subset B$ and $\mu \in \Gamma B$, then $\mu\gamma$ is quasi-enlarging.

Proof.

Let $\lambda \in F$. Since γ is quasi-enlarging, $\gamma(\lambda) \leq \gamma(\lambda \wedge \gamma(\lambda))$. Since $\gamma(\lambda) \in B$ and $\mu \in \Gamma B$, $\gamma(\lambda) \leq \mu(\gamma(\lambda))$ and so $\gamma(\lambda) \leq \gamma(\lambda \wedge \mu\gamma(\lambda))$. Therefore, $\mu\gamma(\lambda) \leq \mu\gamma(\lambda \wedge \mu\gamma(\lambda))$. Hence $\mu\gamma$ is quasi-enlarging.

Theorem 2.6.

Let X be a nonempty set and $\gamma \in \Gamma$. Then $i\gamma$ is quasi-enlarging and $c\gamma$ is enlarging.

Proof.

If $\lambda \in F$, then $i\gamma(\lambda) = i\gamma i\gamma(\lambda) = i\gamma(\lambda \wedge i\gamma(\lambda))$, since $i\gamma(\lambda) \leq \lambda$. So $i\gamma$ is quasi-enlarging. Again, $i\gamma(\bar{}1 - \lambda) \leq \bar{}1 - \lambda$ and so $\lambda = \bar{}1 - (\bar{}1 - \lambda) \leq \bar{}1$

$- i\gamma(\bar{}1 - \lambda) = c\gamma(\lambda)$. Therefore, $c\gamma$ is enlarging.

Theorem 2.7.

Let X be a nonempty set, $\gamma \in \Gamma$ and A be the family of all $\gamma-$fuzzy open sets. If $\mu \in \Gamma$, such that $i\gamma\mu$ is quasi-enlarging and $\kappa \in \Gamma A$, then $\kappa i\gamma\mu$ is quasi-enlarging.

Proof.

If $\lambda \in F$, then $i\gamma\mu(\lambda) \in A$. By Theorem 2.5, it follows that $\kappa i\gamma\mu$ is quasi-enlarging.

Corollary 2.8.

Let X be a nonempty set, $\gamma \in \Gamma$ and A be the family of all $\gamma-$fuzzy open sets. If $\kappa \in \Gamma A$, then $\kappa i\gamma$ is quasi-enlarging.

Proof.

If $\mu : F \to F$ is the identity operator, then $i\gamma\mu = i\gamma$ is quasi-enlarging and so the proof follows from Theorem 2.7.

Let $\{\gamma\iota \in \Gamma \mid \iota \in \Delta \leq = \emptyset\}$ be a family of operations. Define $\phi : F \to F$ by $\phi(\lambda) = \vee\{\gamma\iota(\lambda) \mid \iota \in \Delta\}$ for every $\lambda \in F$. The following Theorem 2.9 gives some properties of ϕ.

Theorem 2.9.

Let X be a nonempty set. Let $\{\gamma\iota \in \Gamma \mid \iota \in \Delta \leq = \emptyset\}$ be a family of operations. Define $\phi : F \to F$ by $\phi(\lambda) = \vee\{\gamma\iota(\lambda) \mid \iota \in \Delta\}$ for every $\lambda \in F$. Then the following hold. (a)$\phi \in \Gamma$. (b) If each $\gamma\iota$ is $B-$enlarging, then so is ϕ.

(c) If each $\gamma\iota$ is quasi-enlarging, then so is ϕ.

(d) If each $\gamma\iota$ is weakly quasi-enlarging, then so is ϕ.

Proof. (a) If $\lambda \leq \nu$, then $\gamma\iota(\lambda) \leq \gamma\iota(\nu)$ and so $\phi(\lambda) = \vee\{\gamma\iota(\lambda) \mid \iota \in \Delta\} \leq \vee\{\gamma\iota(\nu) \mid \iota \in \Delta\} = \phi(\nu)$. Therefore, $\phi \in \Gamma$.

(b) Let $\lambda \in B$. Then, by hypothesis, $\lambda \leq \gamma\iota(\lambda)$ for every $\iota \in \Delta \leq = \emptyset$. Therefore, $\lambda \leq \vee\gamma\iota(\lambda) = \phi(\lambda)$ and so ϕ is $B-$enlarging.

(c) Suppose each $\gamma\iota$ is quasi-enlarging. Then for $\lambda \in F$, $\phi(\lambda) = \vee\gamma\iota(\lambda) \leq \vee\gamma\iota(\lambda \wedge \gamma\iota(\lambda)) \leq \vee\gamma\iota(\lambda \wedge \phi(\lambda)) = \phi(\lambda \wedge \phi(\lambda))$ and so ϕ is quasi-enlarging.

(d) For $\lambda \in F$, $\lambda \wedge \phi(\lambda) = \lambda \wedge (\vee\gamma\iota(\lambda)) = \vee(\lambda \wedge \gamma\iota(\lambda)) \leq \vee\gamma\iota(\lambda \wedge \gamma\iota(\lambda)) \leq \vee\gamma\iota(\lambda \wedge \phi(\lambda)) = \phi(\lambda \wedge \phi(\lambda))$. Therefore, ϕ is weakly quasi-enlarging.

Definition 2.10. Let X be a nonempty set and $A \subset F$. We say that an operation $\gamma \in \Gamma$ is $A-$*friendly*, if $\nu \wedge \gamma(\lambda) \leq \gamma(\nu \wedge \lambda)$ for every $\lambda \in F$ and $\nu \in A$.

The following Example 2.11 gives examples of $A-$*friendly* operators. It is clear that if γ is $A-$*friendly* and $B \subset A$, then γ is a $B-$*friendly* operator. Theorem 2.12 below shows that the composition of friendly operators is again a friendly operator. Theorem 2.13 shows that arbitrary union of friendly operators is again a friendly operator.

Example 2.11. (a) If $\gamma : F \to F$ is defined by $\gamma(\lambda) = \theta$ for every $\lambda \in F$ for some $\theta \in F$, then γ is $A-$friendly for every $A \subset F$. (b) In any fuzzy topological space (X, τ), the fuzzy interior and closure operators $i\tau$ and $c\tau$ are $\tau-$friendly. That is, the following hold. (i)$i\tau(\lambda) \wedge \nu \leq i\tau(\lambda \wedge \nu)$ for every $\lambda \in F$ and $\nu \in \tau$. (ii)$c\tau(\lambda) \wedge \nu \leq c\tau(\lambda \wedge \nu)$ for every $\lambda \in F$ and $\nu \in \tau$.

Theorem 2.12.

Let X be a nonempty set, $\gamma, \gamma 1 \in \Gamma$ and $A \subset F$. If γ and $\gamma 1$ are $A-$friendly operators, then so is $\gamma 1\gamma$.

Proof. Suppose $A \subset F$ such that γ and $\gamma 1$ are $A-$friendly. Then, $\gamma(\lambda) \wedge \nu \leq \gamma(\lambda \wedge \nu)$ for every $\lambda \in F$ and $\nu \in A$, and $\gamma 1(\lambda) \wedge \nu \leq \gamma 1(\lambda \wedge \nu)$ for every $\lambda \in F$ and $\nu \in A$. Replacing λ by $\gamma(\lambda)$ in the second inequality, we get $\gamma 1\gamma(\lambda) \wedge \nu \leq \gamma 1(\gamma(\lambda) \wedge \nu) \leq \gamma 1\gamma(\lambda \wedge \nu)$. Therefore, $\gamma 1\gamma$ is an $A-$friendly operator.

Theorem 2.13. Let X be a nonempty set, $A \subset F$ and $\gamma\iota$ is $A-$friendly for every $\iota \in \Delta$. Then $\phi = \vee\gamma\iota$ is $A-$friendly.

Proof. If $\lambda \in F$, then for $\nu \in A$, $\phi(\lambda) \wedge \nu = (\vee\gamma\iota)(\lambda) \wedge \nu = \vee(\gamma\iota(\lambda)) \wedge \nu = \vee(\gamma\iota(\lambda) \wedge \nu) \leq \vee\gamma\iota(\lambda \wedge \nu) = \phi(\lambda \wedge \nu)$. Therefore, ϕ is an $A-$friendly operator.

Using friendly operators, next we construct quasi-enlarging operators using a generalized fuzzy topology (GFT). Let $\mu \subset F$ be arbitrary. For $\lambda \in F$, define $i\mu(\lambda) = \vee\{\beta \in \mu \mid \beta \leq \lambda\}$ and $i\mu(\lambda) = \bar{0}$, if no such $\beta \in \mu$

exists. Let $\mu^{\leq} = \{\bar{1} - \lambda \mid \lambda \in \mu\}$. Define $c\mu(\lambda) = \wedge\{\beta \in \mu \mid \lambda \leq \beta\}$ and $c\mu(\lambda) = \bar{1}$, if no such $\beta \in \mu^{\leq}$ exists. If μ is the family of all $\gamma-$open sets, then $c\gamma = c\mu$ and $i\gamma = i\mu$.

Theorem 2.14. Let $\mu \subset F$ be a GFT. If $\gamma \in \Gamma$, is $\mu-$friendly, then $i\mu\gamma$ is quasi-enlarging.

Proof. If $\xi \in F$, then $i\mu\gamma(\xi) = \gamma(\xi) \wedge i\mu\gamma(\xi)$. Since $\gamma$is $\mu-$friendly, $\gamma(\xi) \wedge i\mu\gamma(\xi) \leq \gamma(\xi \wedge i\mu\gamma(\xi))$. Therefore, $i\mu\gamma(\xi) = i\mu i\mu\gamma(\xi) \leq i\mu\gamma(\xi \wedge i\mu\gamma(\xi))$ and so $i\mu\gamma$ is quasi-enlarging.

Theorem 2.15. Let $\mu \subset F$ and $\gamma \in \Gamma$ be $\mu-$friendly. If $\nu \in \mu$ and ξ is a $\gamma-$fuzzy open set, then $\xi \wedge \nu$ is again a $\gamma-$fuzzy open set.

Proof. Since ξ is a $\gamma-$fuzzy open set, $\xi \leq \gamma(\xi)$. Then for $\nu \in \mu$, $\nu \wedge \xi \leq \nu \wedge \gamma(\xi) \leq \gamma(\nu \wedge \xi)$ and so $\nu \wedge \xi$ is a $\gamma-$fuzzy open set.

Corollary 2.16. Let $\gamma \in \Gamma$, μ be the family of all $\gamma-$fuzzy open sets and γ be $\mu-$friendly. Then $\lambda \wedge \nu \in \mu$ whenever $\lambda \in \mu$ and $\nu \in \mu$.

Corollary 2.16 leads to define a new subfamily of Γ, namely $\Gamma 4 = \{\gamma \in \Gamma \mid \gamma$ is $\mu\gamma-$friendly$\}$ where $\mu\gamma$ is the family of all $\gamma-$fuzzy open sets. Hence, if $\gamma \in \Gamma 4$, then the GFTS (X, γ) is closed under finite intersection, by Corollary 2.16. We call such spaces as *Quasi-fuzzy topological spaces*. Clearly, if $\gamma \in \Gamma 14$, then $\mu\gamma$ is a fuzzy topological space. The following Example 2.17 shows that $\gamma \in \Gamma 4$ does not imply that $\gamma \in \Gamma 1$.

Example 2.17. Let $X = \mathbf{R}$, the set of all real numbers and F be the family of all fuzzy sets defined on X. Define $\gamma : F \to F$ by $\gamma(\lambda) = \bar{\alpha}$ if $\bar{\alpha} \leq \lambda$, *and* $\gamma(\lambda) = \bar{0}$if otherwise, where $0 < \alpha < 1$. Clearly, $\gamma \leq \in \Gamma 1$. Since $\{\bar{0}, \bar{\alpha}\}$ is the family of all $\gamma-$fuzzy open sets, it follows that $\gamma \in \Gamma 4$.

Theorem 2.18. If X is a nonempty set, F is the family of all fuzzy sets defined on X and $\gamma \in \Gamma 4$, then the following hold. (a) $i\gamma(\lambda \wedge \nu) = i\gamma(\lambda) \wedge i\gamma(\nu)$ for every fuzzy sets $\lambda, \nu \in F$. (b) $c\gamma(\lambda \vee \nu) = c\gamma(\lambda) \vee c\gamma(\nu)$ for every fuzzy sets $\lambda, \nu \in F$. *Proof.* (a) Since $i\gamma(\lambda) \leq \lambda$ and $i\gamma(\nu) \leq \nu$, by Corollary 2.16, $i\gamma(\lambda) \wedge i\gamma(\nu)$ is a $\gamma-$fuzzy open set contained in $\lambda \wedge \nu$ and so $i\gamma(\lambda) \wedge i\gamma(\nu) \leq i\gamma(\lambda \wedge \nu)$. Clearly, $i\gamma(\lambda \wedge \nu) \leq i\gamma(\lambda) \wedge i\gamma(\nu)$. This proves (a). (b) Since $\lambda \vee \nu \leq c\gamma(\lambda) \vee c\gamma(\nu) \leq c\gamma(\lambda \vee \nu)$, it follows that $c\gamma(\lambda \vee \nu) = c\gamma(\lambda) \vee c\gamma(\nu)$ for every fuzzy sets $\lambda, \nu \in F$.

Lemma 2.19. Let $\lambda \in F$, $\gamma \in \Gamma$ and μ be the family of all $\gamma-$fuzzy open sets. Then a fuzzy point $x_t \in c\gamma(\lambda)$ if and only if for every $\mu-$fuzzy open set ν of x_t, $\nu q \lambda$.

Proof. Suppose $x_t \in c\gamma(\lambda)$. Let ν be a $\mu-$fuzzy open set of x_t. If $\nu \tilde{q} \lambda$, then $\lambda \leq (1 - \nu)$. Since $(1 - \nu)$ is $\mu-$fuzzy closed, $c\gamma(\lambda) \leq (1 - \nu)$. Since $x_t \leq \in (1 - \nu)$, $x_t \leq \in c\gamma(\lambda)$, a contradiction. Therefore, $\nu q \lambda$. Conversely, suppose $x_t \leq \in c\gamma(\lambda)$. Since $c\gamma(\lambda) = \wedge\{\xi \mid \lambda \leq \xi$ and ξ is $\mu-$fuzzy closed$\}$, there is a $\mu-$fuzzy closed set $\xi \leq \lambda$ such that $x_t \leq \in \xi$. Then $1 - \xi$ is a $\mu-$fuzzy open sets such that $x_t \in (1 - \xi)$. By hypothesis, $(1 - \xi) q \lambda$. Since $\xi \leq \lambda$, $(1 - \xi) \tilde{q} \lambda$, a contradiction to the hypothesis. Hence $x_t \in c\gamma(\lambda)$.

Theorem 2.20. Let $\lambda \subset F$, $\gamma \in \Gamma$ be $\lambda-$friendly and μ be the family of all $\gamma-$fuzzy open sets. Then $c\gamma$ is $\lambda-$friendly.

Proof. Let $\nu \in \lambda$, $\xi \in F$ and $xt \in \nu \wedge c\mu(\xi)$. If $xt \in \omega \in \mu$, then by Theorem 2.15, $\nu \wedge \omega$ is a $\gamma-$fuzzy open set containing xt. By Lemma 2.19, $(\omega \wedge \nu)q\xi$. Then clearly, $\omega q(\nu \wedge \xi)$ and so $xt \in c\mu(\nu \wedge \xi)$. Hence $\nu \wedge c\mu(\xi) \leq c\mu(\nu \wedge \xi)$ which implies that $c\gamma$ is $\lambda-$friendly.

Corollary 2.21 If X is a nonempty set, F is the family of all fuzzy sets on X, $\gamma \in \Gamma 4$ and μ is the family of all $\gamma-$fuzzy open sets, then the following hold. (a) $c\gamma(\nu) \wedge \xi \leq c\gamma(\nu \wedge \xi)$ for every fuzzy sets $\nu, \xi \in \mu$. (b)$c\gamma(c\gamma(\nu) \wedge \xi) = c\gamma(\nu \wedge \xi)$ for every fuzzy sets $\nu, \xi \in \mu$.

(c) $i\gamma(\nu \vee \xi) \leq i\gamma(\nu) \vee \xi$ for every fuzzy set ν and $\mu-$fuzzy closed set ξ.

(d) $i\gamma(\nu \vee \xi) = i\gamma(i\gamma(\nu) \vee \xi)$ for every fuzzy set ν and $\mu-$fuzzy closed set ξ.

Proof.

(a) The proof follows from Theorem 2.20.

(b). Since $\nu \wedge \xi \leq c\gamma(\nu) \wedge \xi$, the proof follows from (a).

(c) If ξ is $\mu-$fuzzy closed, then $\omega = \bar{1} - \xi \in \mu$ and so by (a), for $\nu \in F$, $c\gamma(\nu) \wedge \omega \leq c\gamma(\nu \wedge \omega)$ and so $\bar{1} - c\gamma(\nu \wedge \omega) \leq \bar{1} - (c\gamma(\nu) \wedge \omega)$. Therefore, $i\gamma((\bar{1} - \nu) \vee (\bar{1} - \omega)) \leq (\bar{1} - c\gamma(\nu)) \vee (\bar{1} - \omega)$ and so $i\gamma((\bar{1} - \nu) \vee \xi) \leq i\gamma(\bar{1} - \nu) \vee \xi$. If $\psi = \bar{1} - \nu$, we have $i\gamma(\psi \vee \xi) \leq i\gamma(\psi) \vee \xi$, which proves (c).

(d) The proof follows from (c).

Corollary 2.22. Let $\lambda \subset F$ be a GFT, $\gamma \in \Gamma$ be $\lambda-$friendly and μ be the family of all $\gamma-$fuzzy open sets. Then $i\mu c\mu$ is quasi-enlarging. Proof. The proof follows from Theorem 2.14 and Theorem 2.20. In the rest of the section, we will consider a special type of enlargement whose domain is a subfamily of F. A function $\kappa : \mu \rightarrow F$ is an *enlargement* if $\lambda \leq \kappa(\lambda)$ for every $\lambda \in \mu$. The following are some examples of enlargements.

Example 2.23. Let X be a nonempty set, F be the family of all fuzzy sets defined on X and $\mu \subset F$. Define $\kappa : \mu \rightarrow F$ by (a)$\kappa(\lambda) = \lambda$ for every $\lambda \in \mu$. (b) $\kappa(\lambda) = c\mu(\lambda)$ for every $\lambda \in \mu$.

(c) $\kappa(\lambda) = i\mu c\mu(\lambda)$ for every $\lambda \in \mu$.

Then κ is an enlargement in each case.

Let $\kappa : \mu \rightarrow F$ is an enlargement. Define $\kappa\mu = \{\lambda \in F \mid$ For each $xt \in \lambda$, there exists $\nu \in \mu$ such that $xt \in \nu \leq \kappa(\nu) \leq \lambda\}$. The following Theorem 2.24 gives some properties of $\kappa\mu$.

Theorem 2.24. Let X be a nonempty set, F be the family of all fuzzy sets defined on X, $\mu \subset F$ and $\kappa : \mu \rightarrow F$ be an enlargement. The the following hold. (a) $\kappa\mu$ is a GFT.

(b) If μ is a GFT, then $\kappa\mu \subset \mu$.

Proof. (a) Clearly,$0 \in \kappa\mu$. Let $\nu\alpha \in \kappa\mu$ for every $\alpha \in \leq$ and $\nu = \vee\{\nu\alpha \mid \alpha \in \leq\}$. If $xt \in \nu$, where $t \in (0, 1]$, then $xt \in \nu\alpha$ for some $\alpha \in \leq$. By hypothesis, there is a $\xi \in \mu$ such that $xt \in \xi \leq \kappa(\xi) \leq \nu\alpha \leq \nu$. Hence $\nu \in \kappa\mu$ which implies that $\kappa\mu$ is a GFT. (b) Let $\nu \in \kappa\mu$. Then for each $xt \in \nu$ where $t \in (0, 1]$, there exists $\xi x \in \mu$ such that $\kappa(\xi x) \leq \nu$ and so $xt \in \xi x \leq \kappa(\xi x) \leq \nu$. Hence $\nu = \vee\{\xi x \mid x \in \nu\}$. Since μ is a GFT, $\nu \in \mu$ and so $\kappa\mu \subset \mu$.

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